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Synchronous asymmetric exclusion processes with extended hopping

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We study the effects of extended hopping in one-dimensional asymmetric simple exclusion processes (ASEP's). One additional parameter v_{max} is introduced and modified ASEP is proposed. Two different phases are observed. The exact values of current and bulk density are obtained. An unusual density fluctuation is observed and discussed.

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I. INTRODUCTION

Recently the dynamical phases of systems driven far from thermal equilibrium have been the subject of extensive studies [1]. The collective behavior shows many interesting phenomena that are absent in thermal equilibrium, such as boundary-induced phase transitions and stationary states characterized by nonvanishing currents. These driving fields are found in a large variety of physics systems such as driven diffusive processes and granular flow. One of the simplest nonequilibrium systems is the fully asymmetric simple exclusion process (ASEP) in one dimension [2]. This model serves as a prototype for nonequilibrium systems and has attracted a lot of interest for its many applications. The model describes a driven lattice gas in one dimension with hard core repulsion, i.e., only the nearest neighbor hopping is considered. The main aim of this paper is to study the ASEP with extended hopping.

The paper is organized as follows: The model is described in the next section. In Sec. III we calculate the current and bulk density. In Sec. IV we analyze the density profiles. The results are summarized in Sec. V with some conclusions and suggestions.

II. MODEL

Consider a one-dimensional lattice of L sites with open boundaries. Particles are introduced into the system at the left end (site 1), move through the bulk, and leave the system at the right end (site L).

Within the bulk, particles move according to deterministic dynamics. Each site is either empty or occupied by a particle with velocity v, which is an integer ranging from 0 to v_{max}. The parameter v_{max} denotes the maximum velocity. The configurations are updated synchronously according to the following successive steps: (1) Acceleration: increase v by 1 if the velocity v is less than the maximum velocity v_{max}. (2) Slowing down: decrease v to d if v is larger than d, which denotes the number of empty sites in front of the particle. (3) Movement: hop v sites forward.

These update rules are also known as the deterministic Nagel-Schreckenberg traffic model [3]. In the case of $v_{max}=1$, the model reduces to the well known ASEP. Particles hop one site to the right if the site in front of them is empty. Instead of the synchronous update considered in this paper, also known as parallel update, the ASEP is usually studied with random sequential update. A comparison of different update procedures can be found in Ref. [4]. With $v_{max}>1$, particles are allowed to hop more than one site within a single time step, provided their velocities are greater than zero and there are enough empty sites in front of them.

The boundary conditions are specified as follows. The two ends of the system are taken as coupled to two stochastic reservoirs. Particles are injected at the left end with probability α, provided the first site (site 1) is empty; particles are removed at the right end with probability β, provided the last site (site L) is occupied. In the following, we simply adopt the constraint that the injected particle has the maximum velocity v_{max}. The model is characterized by three parameters: two rates at the ends (α and β) and the maximum velocity in the bulk (v_{max}). Extended hopping is realized when the maximum velocity is larger than 1.

III. CURRENT AND BULK DENSITY

As the boundary rates α and β change, two different phases can be observed. When the injection rate α is larger than the removal rate β, a high density phase is obtained; when α is less than β, a low density phase results. The phase transition can be observed along the line of equal rates, i.e., $\alpha=\beta$. The exact phase diagrams can be deduced phenomenologically. As the phase transition is induced by the boundary, the relationship between the current J and bulk density ρ is the same as in the case of periodic boundary conditions. This relation is also known as the fundamental diagram. In the case of the high density phase, $\alpha>\beta$, the current is controlled by the removal rate and independent of the injection rate. The fundamental diagram gives $J=1-\rho$, which is independent of the maximum velocity v_{max}. The dependence on v_{max} appears in the onset of this high density region, i.e., $\rho>1/(v_{max}+1)$. The constraint on fixed current on the right end (site L) gives $J=\rho_L\beta$, where ρ_L is the average density on the last site. In the high density phase, particles are crowded together and they are not likely to move with velocity larger than 1. Thus the parameter v_{max} becomes irrelevant and one has $\rho=\rho_L$. The current and bulk density can be solved for easily and are given by

$$J=\frac{\beta}{1+\beta}, \quad \rho=\frac{1}{1+\beta}, \tag{1}$$

which are solely determined by the parameter β.

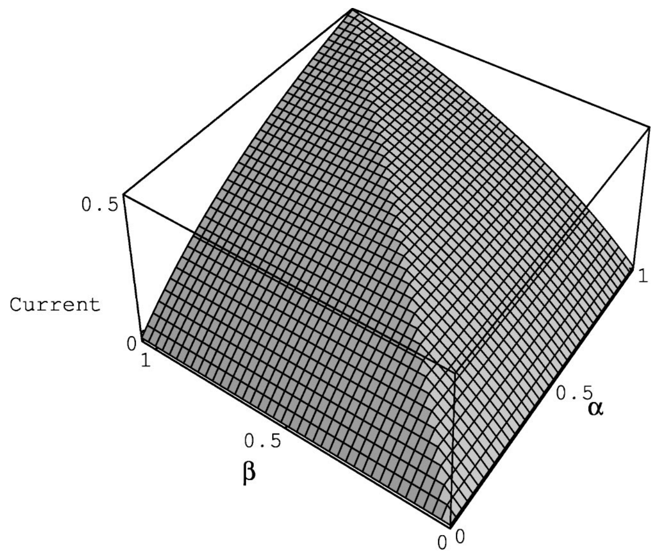

FIG. 1. Current J as a function of rates α and β. The results are independent of v_{max}.

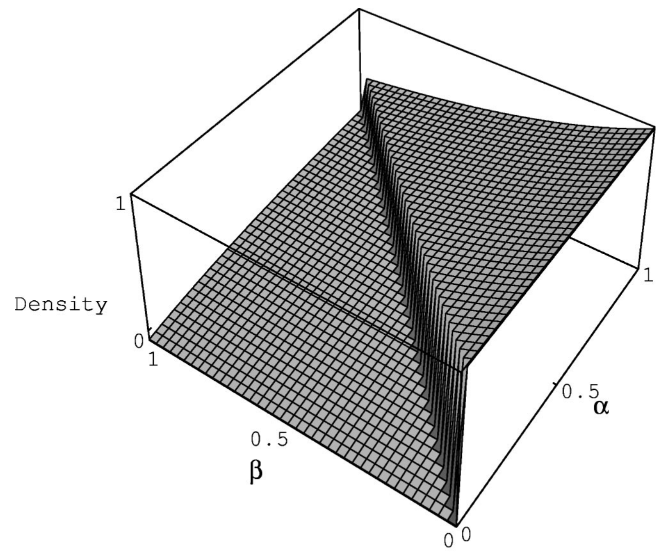

FIG. 2. Bulk density ρ as a function of rates α and β for $v_{max}=2$.

Similarly, in the low density phase, $\alpha<\beta$, the current is controlled by the injection rate and independent of the removal rate. The fundamental diagram gives $J=v_{max}\rho$. The constraint on the fixed current on the left end (site 1) gives $J=(1-\rho_1)\alpha$, where ρ_1 is the average density on the first site. In the low density phase, all particles move with the maximum velocity v_{max}. Thus the bulk density assumes a smaller value and can be expressed as $\rho=\rho_1/v_{max}$. The current and bulk density are then given by

$$J=\frac{\alpha}{1+\alpha},\quad \rho=\frac{1}{v_{max}}\frac{\alpha}{1+\alpha}. \tag{2}$$

It is interesting to note that the current is independent of v_{max}. An increment in the maximum velocity will not lead to a larger value of current, as naively expected. The numerical results are shown in Fig. 1. The distribution is symmetrical between two rates, i.e., $J(\alpha,\beta)=J(\beta,\alpha)$. The results for the bulk density are shown in Fig. 2. A discontinuity of the bulk density is observed along the phase transition line $\alpha=\beta$ and can be expressed as

$$\Delta\rho=\frac{1}{1+\alpha}\left(1-\frac{\alpha}{v_{max}}\right). \tag{3}$$

The only exception is the point $\alpha=\beta=1$ in the case of $v_{max}=1$, which is known to be a critical point of the second order phase transition [2]. The distribution of the high density phase is independent of v_{max} as expected. In the low density phase, the value of the bulk density decreases with increase of v_{max}. Thus the discontinuity of the transition increases with increase of v_{max}.

IV. DENSITY PROFILE

Next, we consider the density profiles. Exact results have been obtained only in the case of $v_{max}=1$, where the density profiles are flat except near the left/right end for the high/low density phase [5].

In the high density phase, particles are inserted much more efficiently than they are removed. Thus the current is determined by the removal rate. To have a fixed current through the bulk, the average density must decrease as one approaches the left end. The average density of the first site can be readily determined from the constraint of fixed current on the left end,

$$\rho_1=\frac{\alpha-\beta+\alpha\beta}{\alpha(1+\beta)}. \tag{4}$$

It is interesting to note that the value of ρ_1 is always smaller than the corresponding bulk density ρ in Eq. (1). Both ρ_1 and ρ are independent of v_{max}. In the case of $v_{max}=1$, as one moves away from the left end, the average density increases monotonically to its asymptotic value, i.e., the bulk density. The exact profile is given by [5]

$$\rho_i=\frac{1}{1+\beta}-\frac{\beta}{\alpha}\frac{1-\alpha}{1+\beta}\left(\frac{\beta}{\alpha}\right)^{i-1}, \tag{5}$$

where ρ_i denotes the average density on the site i. In the case of $v_{max}>1$, the monotonic increase of average density can no longer be observed. The average density fluctuates and can be divided into two branches. The upper branch is followed by sites $i=1$, $v_{max}+1$, $2v_{max}+1$, $3v_{max}+1,\ldots$. The density over the other sites constitutes the lower branch. The numerical results are shown in Fig. 3. As the parameter v_{max} increases, more sites are located on the lower branch. The two branches have the same asymptotic value. Thus the oscillation of the average density can be observed only near the left end.

In the low density phase, the removal of a particle is much more effective than the insertion. The current is determined

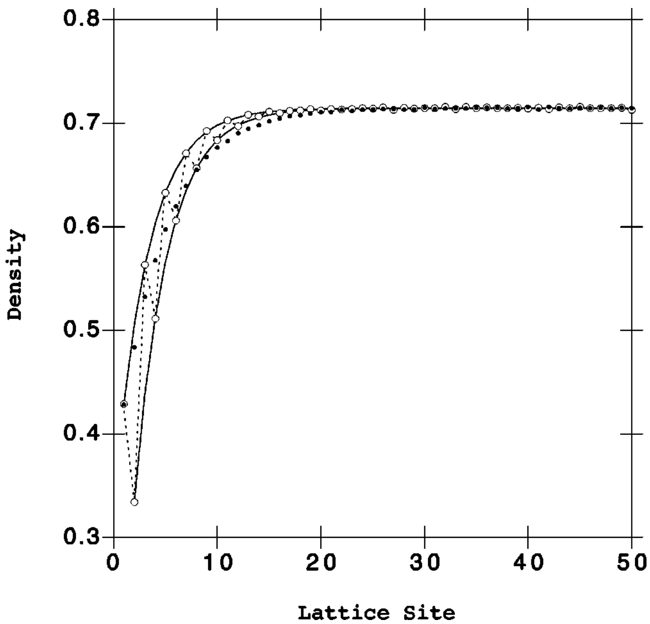

FIG. 3. Density profile ρ_i as a function of site i near the left end for $v_{max}=1$ (filled circles) and 2 (open circles). Parameters $\alpha=0.5$, $\beta=0.4$, and $L=2000$; the high density phase is assumed. The two branches for $v_{max}=2$ are shown by the solid lines. The fluctuation is shown by the dashed line.

by the injection rate and the density profile is expected to be flat except near the right end. However, this can be applied only to the case of $v_{max}=1$.

The average density of the last site can also be determined from the constraint of fixed current on the right end,

$$\rho_L=\frac{\alpha-\beta+\alpha\beta}{\beta(1+\alpha)}, \tag{6}$$

which is always larger than the corresponding bulk density ρ in Eq. (2). Therefore, the average density will increase as one approaches the right end.

In the case of $v_{max}=1$, as one moves away from the right end, the average density decreases monotonically to its asymptotic value, i.e., the bulk density. The exact profile is given by [5]

$$\rho_i=\frac{\alpha}{1+\alpha}+\frac{\alpha}{\beta}\frac{1-\beta}{1+\alpha}\left(\frac{\alpha}{\beta}\right)^{L-i}. \tag{7}$$

When $v_{max}>1$, the average density can also be divided into two branches. In contrast to the case of the high density phase, the fluctuations between these two branches can be observed all through the bulk. The two branches have different asymptotic values. The asymptotic value for the upper branch is $\alpha/(1+\alpha)$, which assumes the same value as in the case of $v_{max}=1$; and the lower branch value decreases to zero. As in the case of the high density phase, these two branches do not have the same number of constituent sites. The ratio is $1/(v_{max}-1)$. As v_{max} increases, more sites are located on the lower branch. The bulk density in Eq. (2) can be readily reproduced as a weighted average of the two asymptotic values. The numerical results are shown in Fig. 4.

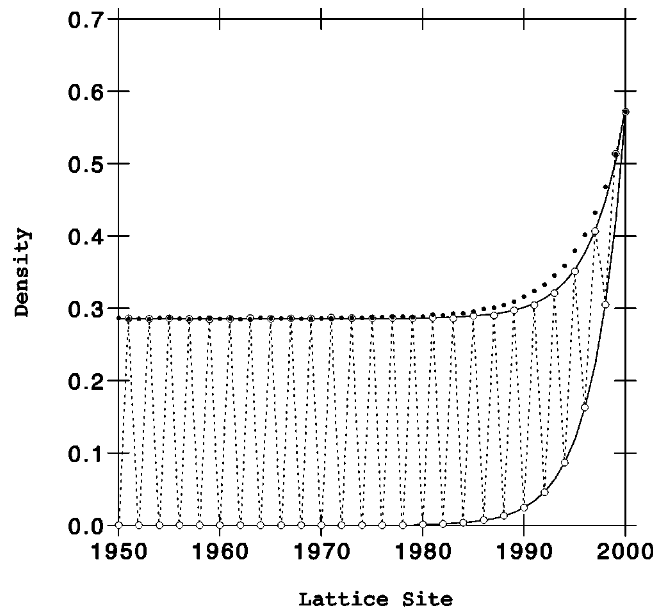

FIG. 4. Density profile ρ_i as a function of site i near the right end for $v_{max}=1$ (filled circles) and 2 (open circles). Parameters $\alpha=0.4$, $\beta=0.5$, and $L=2000$; the low density phase is assumed. The two branches for $v_{max}=2$ are shown by the solid lines. The fluctuation is shown by the dashed line.

In the transition between high density and low density phases, $\alpha=\beta$, a linear density profile is expected. The results are shown in Fig. 5 and are given by

$$\rho_i=\frac{1}{v_{max}}\frac{\alpha}{1+\alpha}+\frac{1}{1+\alpha}\left(1-\frac{\alpha}{v_{max}}\right)\frac{i}{L}. \tag{8}$$

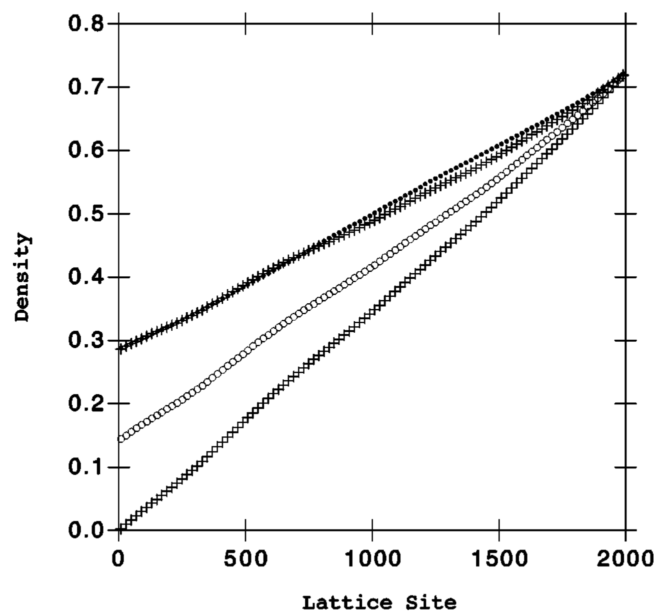

FIG. 5. Density profile ρ_i as a function of site i for $v_{max}=1$ (filled circles) and 2 (open circles for coarse-grained average; crosses for single-site average). Parameters $\alpha=\beta=0.4$ and $L=2000$.

It should be noted that the above results are for the coarse-grained average. If one considers a single-site average, two branches can be observed when $v_{max}>1$. These two branches are both linear and given by

$$\rho_i \ \text{(upper)} = \frac{\alpha}{1+\alpha} + \frac{1-\alpha}{1+\alpha}\frac{i}{L}, \quad \rho_i \ \text{(lower)} = \frac{1}{1+\alpha}\frac{i}{L}. \tag{9}$$

The coarse-grained result in Eq. (8) can be reproduced as the weighted sum of these two branches. In the case of $v_{max}=1$, only the upper branch is present, which gives the same result as in Eq. (8).

V. CONCLUSION

In this paper, we studied the effects of extended hopping in one-dimensional asymmetric simple exclusion processes. One additional parameter v_{max} was introduced to give the possibility for one particle to move by more than one site within a single time step. As in the ASEP, the boundary conditions are stochastic, while the dynamics in the bulk are deterministic.

The phase diagram is the same as in the ASEP. There are two phases resulting from the competition of two rates on the boundaries. When $\alpha>\beta$, particles are inserted much more efficiently than they are removed and a high density phase results. On the contrary, a low density phase is observed when $\alpha<\beta$. The exact values of current and bulk density can be obtained by simple phenomenological considerations. The current is solely determined by one boundary and independent of v_{max}. The boundary with less efficiency determines the current. In the high density phase, the current is controlled by the removal rate; in the low density phase, the current is controlled by the injection rate. As to the bulk density, its value in the high density phase is also independent of v_{max}. In the low density phase, the bulk density decreases with increase of v_{max}.

We also observe some unusual behavior of the density profiles. With nearest neighbor hopping only, $v_{max}=1$, the density profiles are smooth curves. A monotonic behavior is observed between the boundary value and the asymptotic (bulk) one. With extended hopping, $v_{max}>1$, the average densities divide into two branches according to their locations. The smooth behavior is replaced by fluctuations between these two branches. In the high density phase, the two branches have the same asymptotic value and different boundary values. Thus the fluctuations can be observed only as a boundary layer. On the contrary, in the low density phase, the two branches have different asymptotic values and the same boundary value. The fluctuations can be observed all through the bulk, except near the boundary. When the two boundary rates are equal, both the two branches become straight lines. The fluctuations can still be observed. This unusual fluctuation of average density is a characteristic of the deterministic dynamics of extended hopping. It would be interesting to study the effects of stochastic dynamics further.

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